ROUND I: Arithmetic – Percent, interest, discount, fractions
          and decimals

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. How much, in dollars, is 100,000 % of a penny?

2. If we had a 6 % sales tax and you had $20, what is the
    highest priced taxble item that you could pay for and
    not run out of money?

3. Find the single discount which is equivalent to successive
    discounts of 25%, 20%, and 20%.

ANSWERS

(1 pt) 1. _____$_____

(2 pts) 2. _____$_____

(3 pts) 3. _____7o___

Oxford, St.John's, Shrewsbury

ROUND II: Set theory and logic

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. How many subsets of $\{M, A, T, H\}$ have A as an element?

2. For these three sets of real numbers,

   $R = \{x \mid 1 < x < 7\}$, $S = \{x \mid x > 4\}$, and $T = \{x \mid x \geq 2\}$,

   Specify the restriction(s) on x for

   $$(S' \cap R) \cup (T \cap S'),$$

   Where S' denotes the complement of set S.

3. $x, y, z,$ and w are positive numbers, at least two of which are equal. $w + z = y$ and $x + z = w$. List the numbers from left to right in non-decreasing order, using the symbols "$=$" and "$<$" between them.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. $\{x \mid$ _____ $\}$

(3 pts) 3. _____

Auburn, Leicester, Quatoag

ROUND III: Algebra 1 - open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. If the sum of two numbers is 21 and the difference between the two numbers is 11, what is the average of the two numbers?

2. Find two consecutive positive multiples of 3 such that the square of the larger minus 7 times the smaller equals 141.

3. Find all real values of x which satisfy

$$5x + 3\sqrt{x} - 2 = 0$$

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____ and _____

(3 pts) 3. _____

Quaboag, Tantasqua, Westborough

ROUND IV: Sequences and series

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Find the sum of the first 40 terms of the arithmetic sequence $2, 5, \cdots$

2. The numbers $\sqrt[k]{12}$, $\sqrt[3]{12}$, $\sqrt{12}$ form a geometric sequence. Find k.

3. A 7-term sequence starts with 3 and ends with 71. After the first two terms, each term is the sum of the two preceding terms. Find the sum of the five missing terms.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. __ $k =$ _____

(3 pts) 3. _____

Leicester, Millbury, Tahanto

ROUND V: Matrix and determinant operations

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Solve for matrix Y

$$Y + \begin{bmatrix} 3 \\ 6 \end{bmatrix} = -2\begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

2. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 \\ -8 & -11 \end{bmatrix}$, find a 2×2 matrix

   equal to $(A+B)^2$.

3. If $A = \begin{bmatrix} 6 & 2 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ 5 & 0 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$,

   Solve for matrix X in   AX + B = C.

ANSWERS

(1 pt) 1. $Y = \begin{bmatrix} \end{bmatrix}$
_____

(2 pts) 2. $\begin{bmatrix} \end{bmatrix}$
_____

(3 pts) 3. $X = \begin{bmatrix} \end{bmatrix}$
_____

Tahanto, Tantasqua, Worcester Academy

TEAM ROUND: Topics of previous rounds and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM
AND ON THE SEPARATE TEAM ANSWER SHEET   2 points each

1. In selling a car a man expected to make a gain of 20% above what the car had cost him. However, he received $1500 less than he expected and thus had a loss of 5% on the cost of the car. What was the original cost of the car?

2. In a batch of defective toys, 20 have major defects and 14 have minor defects. If there are 25 toys in the batch, how many have both major and minor defects?

3. Solve for x and y $\begin{cases} \frac{1}{3}x - \frac{2}{3}y = -1 \\ .02x + .4y = 2.58 \end{cases}$

4. Find the biggest 3-digit number whose digits form an arithmetic sequence such that the sum of the digits is 12 and the sum of the squares of the digits is 50.

5. If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ and $i^2 = -1$, express A^4 in simplest matrix form.

6. The entries a_1, a_2, a_3, and a_4 in the determinant $\det A = \begin{vmatrix} a_1 & a_4 \\ a_2 & a_3 \end{vmatrix}$ are the first four terms of a geometric sequence.
$\det A$ is equal to -8 times the square of the value of a_2 while $a_5 = 1377$. What is a_1?

7. It is 10:00 o'clock. On my watch the big hand is on 12 and the small hand is on 10. The hands of the watch form an angle. Between what hour and consecutive minutes (like 10:07 and 10:08) is the next time that the hands of the watch form the same measure angle that they formed at 10:00?

8. How old was a person exactly t years ago if exactly x years ago the person was y years old?

9. For what integer n does $n = \sqrt{20+\sqrt{20+\sqrt{20+\sqrt{20+\cdots}}}}$?